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# $0^+$ cross-section ratio in $^{12}\text{Be}(p,t)^{10}\text{Be}$

## Abstract

Using the well-established wave function of  $^{12}\text{Be}(\text{g.s.})$ , I have estimated the cross-section ratio for the reaction  $^{12}\text{Be}(p,t)$  populating the ground-state and first excited  $0^+$  state of  $^{10}\text{Be}$ . I find that the excited-state to ground-state ratio is small for any reasonable value of configuration mixing in  $^{10}\text{Be}$ .

## Disciplines

Physical Sciences and Mathematics | Physics

## Comments

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**$0^+$  cross-section ratio in  $^{12}\text{Be}(p,t)^{10}\text{Be}$** 

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Using the well-established wave function of  $^{12}\text{Be}(\text{g.s.})$ , I have estimated the cross-section ratio for the reaction  $^{12}\text{Be}(p,t)$  populating the ground-state and first excited  $0^+$  state of  $^{10}\text{Be}$ . I find that the excited-state to ground-state ratio is small for any reasonable value of configuration mixing in  $^{10}\text{Be}$ .

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Studies of the  $(p,t)$  reaction on beta-unstable nuclei have recently become possible, using reverse kinematics with radioactive beams incident on a hydrogen target. Results have already appeared for  $^{11}\text{Li}(p,t)^9\text{Li}$  [1] and  $^{14}\text{O}(p,t)^{12}\text{O}$  [2]. Other experiments are probably being planned. In this context, making estimates for the  $^{12}\text{Be}(p,t)^{10}\text{Be}$  reaction seemed worthwhile. Of special interest would be a comparison of cross sections to the  $^{10}\text{Be}$  ground state (g.s.) and the excited  $0^+$  state (exc) at  $E_x = 6.179$  MeV [3]. This latter state is nearly a pure  $^8\text{Be} \times (sd)^2$  excitation [3–5]. An estimate of the mixing of this configuration with the wave function of the  $p$ -shell g.s. of  $^{10}\text{Be}$  yielded a value of about 1% [4,5], using the analog states in  $^{10}\text{B}$ . If  $^8\text{Be}$  were stable, the natural reaction to strongly populate this excited  $0^+$  state would have been  $^8\text{Be}(t,p)^{10}\text{Be}$ . However, it is not obvious that it should be strong in  $^{12}\text{Be}(p,t)$ . I have computed the expected  $(p,t)$  ratio for these two  $0^+$  states as a function of the mixing, and I report those results here.

The  $(p,t)$  reaction can proceed via  $2n$  pair transfer as well as by sequential  $1n$  transfer. In a microscopic treatment, the simultaneous  $2n$  transfer amplitudes for  $A + 2 \rightarrow A$  are obtained by summing over products of  $1n$  transfer amplitudes for  $A + 2 \rightarrow A + 1$  and  $A + 1 \rightarrow A$ . These states in the intermediate nucleus  $A + 1$  and the corresponding  $1n$  transfer amplitudes are identical to those needed for the sequential transfer calculation. For a case such as the one being considered here, the ratios  $(\text{exc/g.s.})_{\text{sim}}$  and  $(\text{exc/g.s.})_{\text{seq}}$  should be about the same [6–9]. One possible exception might be a case of nearly total destructive interference, where a small change to the input of a calculation can cause a large change in the output. The relative phases are also approximately equal, especially if the sequential intermediate channel includes the  $np$   $T = 1$  pair in addition to the deuteron. It might be thought that this  $T = 1$   $np$  pair is unimportant and/or unrealistic. However, consider the selection rule against population of unnatural parity states in  $2n$  transfer. This selection rule is strongly observed in experiments. However, in a sequential calculation, this selection rule is lost if the intermediate channel contains only the deuteron. It is restored if the  $np$   $T = 1$  pair is included.

Satchler [8] stated “Simultaneous and sequential processes tend to yield similar angular distributions and to depend on nuclear structure in the same way.” He explained by noting that if the energies of the important intermediate states do not cover a wide range then their Green’s functions can be replaced by an average. Closure then leads to nuclear overlaps for sequential transfer being identical to those for simultaneous transfer. In the present case, the most important intermediate states are

the first three states of  $^{11}\text{Be}$ , whose energies span a reasonably narrow range. Satchler states “simultaneous and sequential depend in the same way on nuclear bound-state properties.” He also notes that one-step alone works well for *relative* cross sections. He also points out, as I do above, the importance of including the intermediate state containing the  $np$   $T = 1$  pair, and not just the deuteron. Pinkston and Satchler [9] make similar arguments. The expectation that the simultaneous to sequential ratio should be about the same for two  $0^+$  states in the same nucleus is approximately borne out in calculations [10] for the  $^{208}\text{Pb}(p,t)$  reaction to the first two  $0^+$  states of  $^{206}\text{Pb}$ .

The g.s. of  $^{12}\text{Be}$  has become reasonably well determined as a mixture of the two configurations  $^{10}\text{Be}(p\text{-shell g.s.}) \times (sd)^2$  and  $^{12}\text{Be}(p\text{-shell g.s.})$ . The  $^{10}\text{Be}(t,p)$  reaction [11] was instrumental in demonstrating that the g.s. of  $^{12}\text{Be}$  contains a large  $(sd)^2$  component. A calculation [12] of the  $^{12}\text{Be}$ – $^{12}\text{O}$  mass difference, plus a simple  $(sd)^2$  shell-model calculation, produced a wave function containing 68%  $(sd)^2$  and 32%  $p$  shell that has provided remarkable agreement with a variety of processes involving  $^{12}\text{Be}(\text{g.s.})$ . These processes include beta decay [13], Coulomb energy [12], neutron removal [14],  $^{14}\text{C}(p,t)$  [15–17],  $^{14}\text{O}(p,t)$  [2,18,19],  $\text{B}(E2)$  [19–21], and Gamow-Teller transitions from  $^{12}\text{B}$  [22]. A summary is given elsewhere [23]. The  $^{12}\text{Be}(p,t)$  reaction would allow a further test of that wave function and of those in  $^{10}\text{Be}$ .

With this  $^{12}\text{Be}(\text{g.s.})$  wave function, and with no mixing between the two  $^{10}\text{Be}$   $0^+$  configurations, the  $^{12}\text{Be}(p,t)$  reaction to the excited  $0^+$  state involves pickup only from the  $p$  shell from the  $^{10}\text{Be} \times (sd)^2$  component of  $^{12}\text{Be}(\text{g.s.})$ . This is because the  $p$ -shell  $^{12}\text{Be}$  component contains no  $(sd)^2$  neutrons that are present in the  $^8\text{Be} \times (sd)^2$   $0^+$  excitation.

In the  $^{10}\text{Be}(t,p)$  reaction [11], distorted-wave Born approximation (DWBA) calculations indicated that the cross section for the relevant  $(sd)^2$  transfer was about four times that for transfer into a pure  $p$ -shell  $^{12}\text{Be}(\text{g.s.})$ , using  $p$ -shell  $2n$  transfer amplitudes from Cohen and Kurath [24]. The observed g.s. cross section was about five times that for the computed transfer into the pure  $p$ -shell g.s. Of course, the nuclear structure amplitudes for  $^{10}\text{Be}(t,p)^{12}\text{Be}(\text{g.s.})$  are identical to those for  $^{12}\text{Be}(p,t)^{10}\text{Be}(\text{g.s.})$ . Even for a pure  $p$ -shell  $^{10}\text{Be}(\text{g.s.})$ , these involve both  $(sd)^2$  and  $p$ -shell transfer.

In a two-state model, we can write the g.s. of  $^{10}\text{Be}$  as

$$\text{g.s.} = a \, ^{10}\text{Be}(p\text{-shell g.s.}) + b \, ^8\text{Be}(p\text{-shell g.s.}) \times (sd)_0^2$$

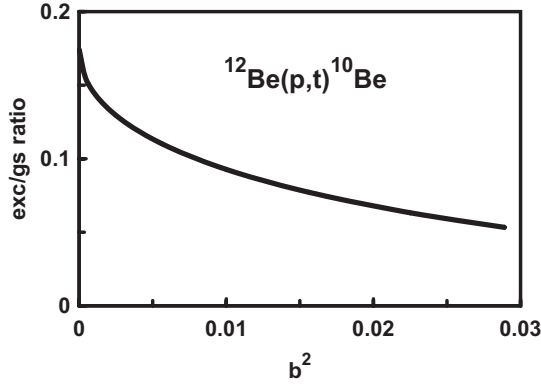


FIG. 1. For the reaction  $^{12}\text{Be}(p,t)^{10}\text{Be}$ , the expected cross-section ratio for the excited  $0^+$  state at 6.18 MeV to the g.s. is plotted vs  $b^2$ , the  $^8\text{Be} \times (sd)^2$  intensity in  $^{10}\text{Be}(\text{g.s.})$ .

and the excited  $0^+$  state as

$$\text{exc} = -b^{10}\text{Be}(p\text{-shell g.s.}) + a^{8}\text{Be}(p\text{-shell g.s.}) \times (sd)_0^2.$$

Then with

$$^{12}\text{Be}(\text{g.s.}) = \alpha^{10}\text{Be}(p\text{-shell g.s.}) \times (sd)^2 + \beta^{12}\text{Be}(p\text{-shell g.s.})$$

we have the transfer amplitudes:

$$\begin{aligned} A(\text{exc}) &= \alpha a A(^{10}\text{Be} \rightarrow ^8\text{Be}) - \alpha b A[(sd)^2] \\ &\quad - \beta b A(^{12}\text{Be} \rightarrow ^{10}\text{Be}), \\ A(\text{g.s.}) &= \alpha b A(^{10}\text{Be} \rightarrow ^8\text{Be}) + \alpha a A[(sd)^2] \\ &\quad + \beta a A(^{12}\text{Be} \rightarrow ^{10}\text{Be}), \end{aligned}$$

where the amplitudes with listed nuclei are all for pure  $p$ -shell ground states. In Cohen and Kurath [24], the  $L = 0$   $2n$  cluster transfer spectroscopic factor is designated as SMAG. For  $^{10}\text{Be} \rightarrow ^8\text{Be}$  SMAG is 0.889, and for  $^{12}\text{Be} \rightarrow ^{10}\text{Be}$  it is 0.786.

Thus, the amplitude  $A(^{10}\text{Be} \rightarrow ^8\text{Be})$  is about 1.13 times the amplitude  $A(^{12}\text{Be} \rightarrow ^{10}\text{Be})$ . The  $(sd)^2$  wave function [12] has intensities of 78%  $s^2$  and 22%  $d^2$ . As stated above, the cross section calculated with this  $(sd)^2$  wave function is about four times that for the  $p$ -shell  $^{12}\text{Be}$ .

Then, with  $r = A(\text{exc})/A(\text{g.s.})$ , and  $\alpha$  and  $\beta$  from above, I have plotted  $r^2$  vs  $b^2$  in Fig. 1. The ratio is 0.17 for no mixing in  $^{10}\text{Be}$ , and it decreases smoothly as the assumed mixing is increased. This plot is for the nuclear structure amplitudes only. Kinematic effects would be included in a DWBA calculation. Because of the large difference in  $Q$  values, I would expect the excited  $0^+$  state to be even more hindered than indicated in the figure. The primary reason arises from the fact that pickup to an excited state involves larger binding energy, leading to a smaller wave function in the exterior, where most of the reaction takes place. Thus, this curve should serve as an upper limit to the cross-section ratio.

If the  $^{12}\text{Be}(p,t)$  reaction is performed, care should be taken to separate the contribution of the  $1^-$  state at 5.96 MeV. This  $1^-$  state could be quite strong in this reaction. The  $2^-$  state at 6.26 MeV should be much less of a problem, because of its unnatural parity.

It thus appears that the first excited  $0^+$  state in  $^{10}\text{Be}$  will be weak in the reaction  $^{12}\text{Be}(p,t)$ . Of course, the structure of that state is such that it should be quite strong in the reaction  $^8\text{Be}(t,p)$ , but (alas) that reaction is not possible.

*Note added:* Recently, an arXiv preprint has appeared [25], using the concept of pairing vibrations to compute cross sections and their ratios. For the ratio being considered here, their prediction is a value of about 2.3, in strong disagreement with my estimate of about 0.1. The quantities making up the ratios are not exactly the same. Theirs is the ratio of cross sections integrated over a fixed angular range, whereas mine is a ratio of  $2n$  transfer strengths. I mentioned above that, because of  $Q$ -value effects, the cross section of the excited state could very well be smaller than that indicated by my ratio. I encourage an experiment to settle the issue.

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